Exercise 28

If $g(x) = x^4 - 2$, find $g'(1)$ and use it to find an equation of the tangent line to the curve $y = x^4 - 2$ at the point $(1, -1)$.

Solution

Determine the derivative of $g(x)$.

$$\begin{aligned} g'(x) &= \lim_{h\to 0} \frac{g(x+h) - g(x)}{h} \\ &= \lim_{h\to 0} \frac{[(x+h)^4 - 2] - [x^4 - 2]}{h} \\ &= \lim_{h\to 0} \frac{[(x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4) - 2] - x^4 + 2}{h} \\ &= \lim_{h\to 0} \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h} \\ &= \lim_{h\to 0} (4x^3 + 6x^2h + 4xh^2 + h^3) \\ &= 4x^3 \end{aligned}$$

Plug in $x = 1$ to this formula to get $g'(1)$.

$$g'(1) = 4(1)^3 = 4$$

This is the slope of the tangent line to the curve at $x = 1$. Use the point-slope formula and the provided point $(1, -1)$ to get the equation of this line.

$$y - (-1) = 4(x - 1)$$

$$y + 1 = 4x - 4$$

$$y = 4x - 5$$

Below is a graph of the curve along with the tangent line at $x = 1$.